ADMISSION TO THE DOTTORATO — PISA — OCTOBER 2012

The applicant should discuss at least three of the following five problems and write a unique short (no more than one page) essay on one of the three proposed topics. The constants or the variables —which are not mentioned in the text— that are possibly needed to solve the problems should be introduced by the applicant, who should also mention explicitly the assumptions that are made. Justify the answers.

1. When the revolving horizontal overhead rotors, of length L, of a helicopter give a downward velocity v to the air, the helicopter can maintain zero velocity in time. Estimate the mass m of the helicopter.

1. $m \approx$

2. Two bodies with the same heat capacity $C(T)$, which depends on temperature according to the law $C(T) = aT$ (with $a > 0$), initially have temperatures T_1 and T_2, with $T_1 > T_2$. After the bodies are put in thermal contact, they reach the equilibrium temperature T_{eq}. Compute T_{eq} and the variation ΔS of entropy of the process (neglect heat transfers of the bodies with the ambient).

2. $T_{eq} =$ $\Delta S =$

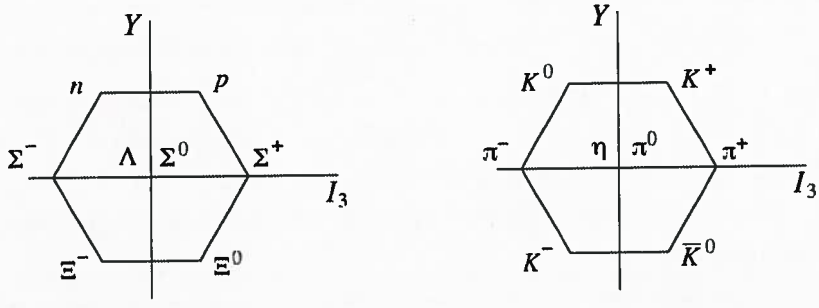
3. Produce the expression of the internal energy $U(T, V)$ of a photon gas in thermal equilibrium with temperature T and volume V (possible numerical constants do not need to be computed).

3. $U(T, V) =$

4. A point P, with cartesian coordinates $\boldsymbol{x}_0$, is contained in the interior of a non conducting body with a generic shape which has a nontrivial uniform electric charge density inside its volume. Let $\boldsymbol{E}(\boldsymbol{x}_0)$ denote the value of electric field vector in P. The body is now modified by the introduction of a spherical cavity, centered in P, which is entirely contained inside the body; the charge density in the remaining part of the body is not changed. Is the new electric field vector $\boldsymbol{E}'(\boldsymbol{x}_0)$ —of the body with the cavity— different from $\boldsymbol{E}(\boldsymbol{x}_0)$? Explain why.

4. $\boldsymbol{E}'(\boldsymbol{x}_0)$ is

5. The particle multiplets of barions and mesons are shown in the figure.



Which of the following strong reactions are forbidden?

(1) $p + \pi^- \to \Sigma^+ + K^-$

(2) $n + \pi^0 \to \Sigma^+ + \pi^-$

5. The forbidden reactions are

PROPOSED TOPICS

[1] The measure of a fundamental constant in Physics; in particular, discuss the accuracy limits (for example, systematic errors) and precision of the measure (for example, reproducibility).

[2] The theory of symmetry in quantum mechanics.

[3] Neutrino oscillations.

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1. In certain conditions, a biatomic molecule can be described by system composed by two point-like atoms of masses m_1 and m_2 (with $m_1 \neq m_2$) at a fixed distance D. Write the expression of the kinetic energy E_k of the molecule in the system in which the center of mass is at rest.

1. $E_k =$

2. A container which is thermally isolated is composed of two parts; one compartment contains one mole of a biatomic perfect gas whereas the second compartment contains two moles of a monoatomic perfect gas. Both gases have the same temperature and the same pressure. At a given moment, the partition wall is removed. In the transition from the initial state to the equilibrium state, what is the variation ΔS of entropy ?

2. $\Delta S =$

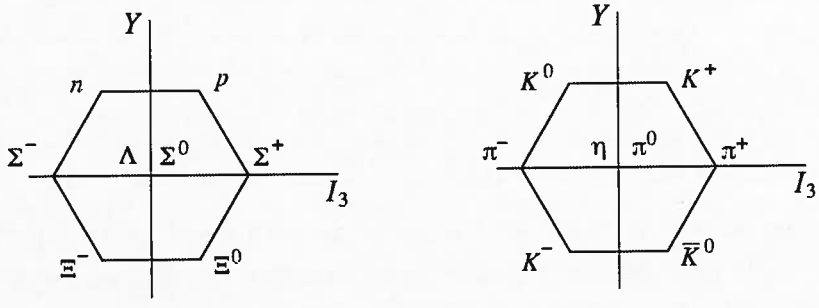
3. Consider a system of noninteracting identical particles that satisfy an "intermediate" statistics in which, for each one-particle state $|\alpha\rangle$ (with $\alpha = 1, 2, ...$) with energy ϵ_α, the number of particles in this state can assume only the following three values $0, 1, 2$. Derive the mean value $\langle n_\alpha \rangle$ of the number of particles in the state $|\alpha\rangle$, as a function of the temperature T and of the chemical potential μ.

3. $\langle n_\alpha \rangle =$

4. Two conducting spheres of radius R have electric charges $Q_1 = Q$ e $Q_2 = 0$. The spheres are sufficiently far away one from the other so that the mutual electrostatic induction can be neglected. At a given time, the spheres are connected by means of a conducting cable with finite electric resistance (the emission of electromagnetic radiation can be neglected). Compute the energy E which is thermally dissipated by the resistance.

4. $E =$

5. The particle multiplets of barions and mesons are shown in the figure.



Which of the following strong reactions are forbidden?

(1) $n + \pi^+ \to \Sigma^+ + \pi^0$

(2) $\Xi^0 + \pi^0 \to \Lambda + K^0$

5. The forbidden reactions are

PROPOSED TOPICS

[1] Methods for the reduction of noise in Physics.

[2] The semiclassical approximation in quantum mechanics.

[3] Parity violations in Physics.

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1. At a given instant, the distance of a asteroid from the sun is equal to R; the asteroid has velocity $\boldsymbol{v} = v_1\boldsymbol{n}_1 + v_2\boldsymbol{n}_2$ (with $v_1 > 0$ and $v_2 > 0$), in which the unit vector $\boldsymbol{n}_1$ is directed from the asteroid to the center of the sun and the unit vector $\boldsymbol{n}_2$ is orthogonal to $\boldsymbol{n}_1$, i.e. $\boldsymbol{n}_1 \cdot \boldsymbol{n}_2 = 0$. Find the minimal distance d of the asteroid from the sun.

1. $d =$

2. One mole of a bi-atomic perfect gas undergoes a irreversible transition in which the pressure remains constant whereas the volume gets reduced by half, that is $V_{final} = V_{initial}/2$. Compute the entropy variation ΔS.

2. $\Delta S =$

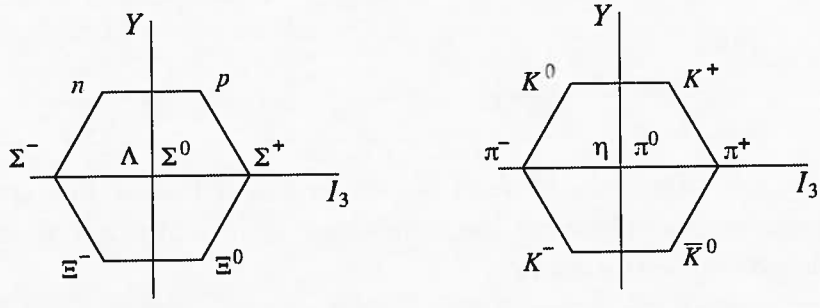
3. Show that, in the absence of an external magnetic field, the ordered state of a sufficiently long ferromagnetic one-dimensional chain is not stable. To this end, starting from the initial completely ordered configuration consider the free energy variation ΔF which is induced by a one kink formation. The kink is due to the inversion of the direction of all the magnetic dipoles that are placed on the right of a given point in the chain. Let ΔE be the energy cost for the formation of one kink in a generic random position among the N sites of the chain. Compute ΔF and show that $\Delta F < 0$ for a sufficiently long chain.

3. $\Delta F =$ $\Delta F < 0$ if ...

4. Consider a beam of polarized photons, with fixed frequency ν and polarization specified by the vertical unit vector $\boldsymbol{z}$. The ray is incident on a polaroid whose principal axis is directed as the unit vector $\boldsymbol{n}$ which forms an angle $\pi/6$ with $\boldsymbol{z}$. Let I_0 be the intensity of the incident ray and let I be the intensity of the transmitted ray through the polaroid (the intensity is given by the energy transported by the photon beam per unit time and per unit surface orthogonal to the ray). Find the value of the ratio $R = I/I_0$.

4. R =

5. The particle multiplets of barions and mesons are shown in the figure.



Which of the following strong reactions are forbidden?

(1) $p + \pi^0 \to \Lambda + \pi^+$

(2) $n + K^- \to \Xi^0 + \pi^-$

5. The forbidden reactions are

PROPOSED TOPICS

[1] The most important kinds of noise in Physics.

[2] Selection rules for the electromagnetic transitions in atomic systems.

[3] Flavor symmetries of strong interactions.